Schoolworkout
Maths

Introduction to Probability: Lesson Notes and Examples

Key words-

Experiment Outcome Event

Example
Experiment: Pick a card from a standard pack of 52 cards
Outcomes: 2 of hearts, 9 of clubs, ace of spades, etc.
Event: picking a card with an even number on.

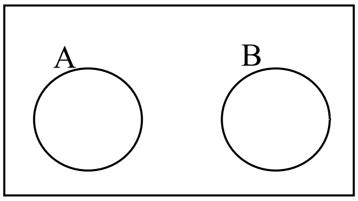
P(even number) = $\frac{20}{52} = \frac{5}{13}$ (2, 4, 6, 8, or 10 of each suit)

Definition: An **event** is a set of possible outcomes.

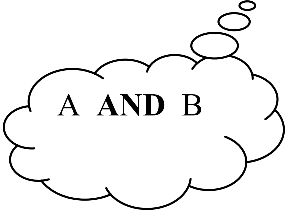
Key results
Let A and B be two events.

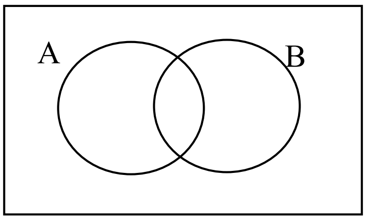
- ✓ $0 \le P(A) \le 1$ Note: P(A) is short for the probability of A.
- ✓ P(A not happening) = 1 – P(A) Note: A′ means *not* A
- ✓ If A and B are *mutually exclusive* (so that they cannot occur at the same time), then
  P(A or B) = P(A) + P(B)

  N.B. $P(A \cup B)$ means P(A or B)



- ✓ More generally, if A and B are not mutually exclusive:
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$





Example: A university has 4000 first year undergraduates
2800 live in halls
1000 live in privately-rented accommodation
500 study medicine
340 of the medical students live in halls.
A student is picked at random.

Let H be the event the student lives in halls; R in privately-rented accommodation; M that they study medicine. Then the above information can be represented by the probabilities:

$P(H) = \frac{2800}{4000} = \frac{7}{10}$ $P(R) = \frac{1000}{4000} = \frac{1}{4}$

$P(M) = \frac{500}{4000} = \frac{1}{8}$ $P(M \cap H) = \frac{340}{4000} = \frac{17}{200}$

So, $P(H \cup R) = \frac{7}{10} + \frac{1}{4} = \frac{19}{20}$ (you can add the probabilities since H and R are mutually exclusive – you can't live in both types of accommodation at the same time).

But $P(H \cup M) = P(H) + P(M) - P(H \cap M)$

$= \frac{7}{10} + \frac{1}{8} - \frac{17}{200} = \frac{37}{50}$ (H and M are not mutually exclusive – you can be a medical student and live in halls).

Independent events

A and B are *independent* if the probability of one happening isn't affected by whether the other happens or not. If A and B are independent, then:

$$P(A \cap B) = P(A) \times P(B).$$

Example: A dice is thrown twice. Find the probability

a) of getting a 4 on both occasions;
b) that neither number is a 2;
c) both numbers are the same.

a) When you throw a dice twice, the outcomes of the throws are independent of each other. So

$$P(4 \text{ AND } 4) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

b) $P(\text{not } 2 \text{ AND not } 2) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

c) $P(\text{both numbers are the same}) = P(1, 1) + P(2, 2) + \ldots + P(6, 6) = \frac{1}{36} + \frac{1}{36} + \ldots + \frac{1}{36} = \frac{1}{6}$

Some key techniques:

Sample space diagrams:

Eg two dice are each numbered 1, 2, 2, 3, 3, 6. They are both thrown and their scores are added together.

There are 36 equally likely combinations of scores:-

Second dice

First dice

	1	2	2	3	3	6
1	2	3	3	4	4	7
2	3	4	4	5	5	8
2	3	4	4	5	5	8
3	4	5	5	6	6	9
3	4	5	5	6	6	9
6	7	8	8	9	9	12

$$\text{P(total is 5)} = \frac{6}{36} = \frac{1}{6}$$

$$\text{P(total is even)} = \frac{18}{36} = \frac{1}{2}$$

$$\text{P(total score is at least 6)} = \frac{15}{36} = \frac{5}{12}$$

Tree diagrams

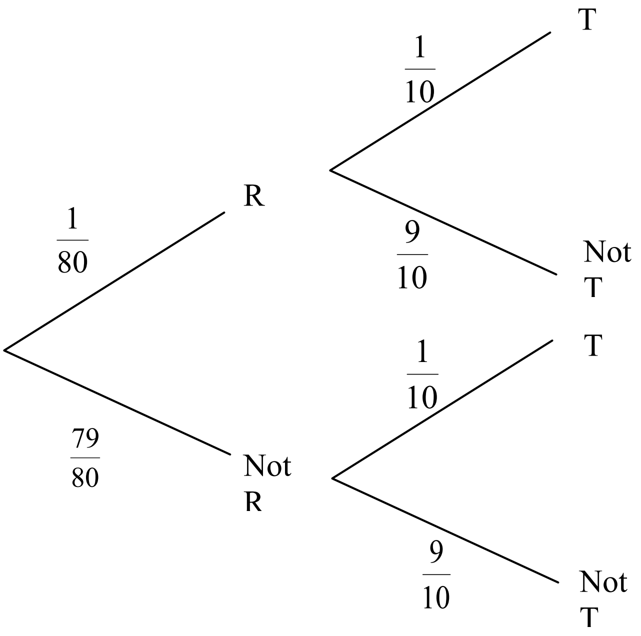
Eg Lydia buys a ticket in her school fete's raffle and tombola. The probability that she wins a prize in the raffle is 1/80 and in the tombola is 1/10.

Find the probability that she wins:

a) 2 prizes;

b) at least one prize.

R = wins prize in raffle, T = wins prize in tombola.



The events of winning a prize in the raffle and in the tombola are independent of each other. Therefore:

a) P(2 prizes) = P(R AND T) = $\frac{1}{80}\times\frac{1}{10}=\frac{1}{800}$

b) P(at least one prize) = 1 – P(no prizes) = $1-\frac{79}{80}\times\frac{9}{10}=\frac{89}{800}$